International Journal of Mine Water, Vol. 3 (2), (1984) 33-54
Printed in Granada, Spain

AN APPRAISAL OF MATHEMATICAL MODELS TO PREDICT WATER INFLOWS INTO UNDERGROUND COAL WORKINGS

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ABSTRACT

The paper describes various analytical and numerical models which have been used to predict the quantities of water flowing into underground mines. Analytical models of flow into wells are discussed briefly and their limitations when applied to mine water flow are listed. Numerical models designed specifically for underground mines are examined in detail under the three categories of water resource models, mine water models and methane and water models. The extent to which the available numerical models are applicable to British longwall coal mining operations is considered and the need for further work is identified.

INTRODUCTION

Water is a major consideration in the planning of any surface or underground mine not only because it is a physical hindrance to the extraction of minerals but also because mining activities frequently degrade important water resources. Water occurrences in underground mines range from sudden and dramatic inrushes to steady inflows of nuisance water. Consequent water problems are as follows.

1) Mine water inrushes can cause the loss of life: 258 people died in United Kingdom coal mines between 1900 and 1980 as a result of inrushes [1].

2) Water inrushes can delay or cause the permanent cessation of mining work together with the loss of equipment [2]. The enourmous capital investment required by mining operations means that not only is the cost of abandoning equipment high but delays in working are also expensive.

3) Steady inflows into a mine require water to be pumped out because mining cannot proceed under waterlogged conditions [3].

4) Mining activities often affect water resources by pollution [4] although this is not a serious problem with British coal mines [5].

5) Mining activities can result in the lowering of water levels and pressures in aquifers, thus threatening the supply of good quality water [4], [6] and [7].

The Need for a Predictive Tool

The ability to predict water flow rates is an important aid to mine planning. A knowledge of likely water quantities contributes to an assessment of the economic viability of a proposed mine. Forecast water flow rates are used to determine the pumping capacity required by an operating mine; an accurate forecast results in a reduction in the number of costly delays caused by unexpectedly large flow rates. Predicted water flow patterns are part of the information needed to select the best of several possible approaches to a mining problem on the basis of the operational difficulty and environmental impact of each.

Mathematical modelling provides the necessary tool for predicting the flow of water into mines.

MATHEMATICAL FORMULATION

The two equations which are usually taken to govern groundwater flow are Darcy's Law and a mass conservation equation. Darcy's Law, which is described in more detail by Scheidegger [8] is inferred from the results of experiments on the steady flow of water through homogeneous isotropic media. In its simplest form it relates a flux or seepage velocity of water through a porous medium to the applied hydraulic pressure gradient. Extended to describe the flow of any fluid through a heterogeneous and anistropic porous medium Darcy's Law can be stated as

$$\underset{\sim}{u} = -\underset{\sim}{K}\,\underset{\sim}{\nabla}\, h \qquad (1)$$

where $\underset{\sim}{u}$ and h are the seepage velocity and head of the fluid and $\underset{\sim}{K}$ is the conductivity of the porous medium. The fluid head is defined in terms of the pressure p and density ρ of the fluid according to the equation

$$h = \frac{p}{\rho g} + z \qquad (2)$$

in which z is the elevation above some fixed datum line and g is the acceleration due to gravity. The conductivity $\underset{\sim}{K}$ is an indication of the ability of the porous medium to transmit fluid and is a property of the fluid as well as the porous medium. This dependence can be separated by writing

$$\underset{\sim}{k} = \frac{\mu}{\rho g}\,\underset{\sim}{K} \qquad (3)$$

where μ is the fluid viscosity and $\underset{\sim}{k}$ is the specific permeability of the porous medium.

Anisotropic Effects

The orientation of rock strata, and the cracks and fissures in them, may lead to anisotropic phenomena which can be described by writing the conductivity as a tensor quantity

$$\underset{\sim}{K} = \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \quad (4)$$

in which K_{xx} etc are scalars. The component of seepage velocity in the x direction is given by

$$u_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z} \quad (5)$$

and the other two components are given by similar expressions. The fluid flow and pressure gradient can be parallel only along specific directions known as the three principal axes of conductivity. If co-ordinate axes are chosen along these directions the only non-zero entries in the conductivity tensor are the diagonal terms which define the principal conductivities. A further simplification is achieved if the medium is isotropic when all the principal conductivities are equal and Darcy's Law reduces to

$$\underset{\sim}{u} = -K \underset{\sim}{\nabla} h \quad (6)$$

Limitations to Darcy's Law

Darcy's Law does not strictly apply to flow in cracks but it is generally assumed that average conductivities can be defined over fairly large volumes so that cracked rocks can be treated as standard porous media. Neither is Darcy's Law valid when the seepage velocity of the fluid is greater than some critical value. Attempts to quantify this critical velocity have not been successful and most workers assume that actual velocities near mining operations are lower than the critical velocity. Equations other than Darcy's Law have been used by workers such as Pérez-Franco [9] to describe the non-linear flow of groundwater. Newman [10] has derived a transient version of Darcy's Law in which a fluid acceleration term is included, however it can be argued that this time dependent effect is negligible for typical mining situations.

The Mass Conservation Equation

The principle of mass conservation requires the total rate at which fluid enters any volume of porous medium to be equal to the rate at which it is stored. The mathematical statement of this requirement has been given by Bear [11] as

$$\underset{\sim}{\nabla} \cdot (\rho \underset{\sim}{u}) = -\frac{\partial}{\partial t}(\rho n) \qquad (7)$$

where n is the effective porosity of the medium and t is the time. Equation (7) can be expanded to give

$$\rho \underset{\sim}{\nabla} \cdot \underset{\sim}{u} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla} \rho = -\rho \frac{\partial n}{\partial t} - n \frac{\partial \rho}{\partial t}. \qquad (8)$$

According to Bear [11] the spacial variations in fluid density are much smaller than the time variations for many practical situations so that

$$\underset{\sim}{u} \cdot \underset{\sim}{\nabla} \rho \ll n \frac{\partial \rho}{\partial t} \qquad (9)$$

and can be ignored. If the fluid is incompressible then equation (8) can be reduced to the simpler form

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{u} = -\frac{\partial n}{\partial t} \qquad (10)$$

Some workers relate changes in porosity to fundamental material properties such as Young's modulus of elasticity. It is more usual to assume that porosity changes solely as a result of variations in fluid head and to assume a linear relationship between them so that equation (10) becomes

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{u} = -s \frac{\partial h}{\partial t} \qquad (11)$$

in which s is the specific storage of the porous medium. If the medium is incompressible its specific storage is zero and equation (11) becomes

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{u} = 0 \qquad (12)$$

which is independent of time and is also used for steady state solutions in which all transient effects have decayed.

Model Formulation

Combining the two governing equations gives a single partial differential equation for the fluid head or alternatively for its pressure. The transient forms of the equations for an incompressible fluid are

$$\underset{\sim}{\nabla} \cdot (\underset{\sim}{K} \underset{\sim}{\nabla} h) = s \frac{\partial h}{\partial t} \qquad (13)$$

or $$\underset{\sim}{\nabla} \cdot \left(\frac{\underset{\sim}{k}}{\mu} \underset{\sim}{\nabla} [p + \rho g z]\right) = s \frac{\partial}{\partial t}\left(\frac{P}{\rho g}\right) \qquad (14)$$

while the steady state forms are given if the right hand sides are replaced by zeros. More complicated forms of the governing partial differential equation describe the flow of compressible fluids such as air and methane. Seepage velocities are obtained from Darcy's Law once the spacial distribution of head or pressure is known.

Different mathematical models of groundwater flow are distinguished by the shape of the boundary and the nature of the boundary conditions. For mining operations the boundary includes sources of water and the mined regions and is usually completed by sections sufficiently far from the mining operation to be unaffected by it. The conditions on the source boundaries and the mine or water sink boundaries are usually known water pressures. The positions of water sources can be hard to define, particularly for a shallow mine where they may extend as high as the ground surface. Under this circumstance the position of the water table has to be calculated, often as a function of time, and the region of unsaturated flow above it may be influenced by atmospheric effects such as rainfall and evaporation. The source boundary for a deep mine is likely to be an aquifer with characteristics which are independent of surface effects.

Simplifying Assumptions

In order to predict water flow rates the governing equation must be solved within the domain defined by the boundaries to give water head or pressure as a function of position and time. Before a solution of a model is attempted a search is made for simplifying modifications. The aim is to make the model as straightforward as possible without seriously jeopardising its ability to describe actual mining operations. Simplifications are sought in the geometry of the model, the governing equation and the boundary conditions.

It is usually possible to reduce a three dimensional geometry to two dimensions if important features are sufficiently constant in one direction. It is also commonly assumed that sources are circular or of infinte extent, that strata are horizontal, that the mine can be represented by a circular opening and that the problems are radially symmetric. A possible simplification in the governing equation uses the steady state rather than the transient form. The material properties of rock and water are often taken as constant in space and time while the rock conductivity is further assumed to be homogeneous. Assumptions about the boundary conditions generally concern the source and mine boundaries. The precise behaviour of these is rarely known so conditions which are simple and intuitively reasonable are applied.

Water flow models which are based on Darcy's Law, or a non-linear variation of it, and a mass conservation equation are collectively referred to as deterministic models. Stochastic models based on probability and past experience, such as those described by Rogoz and Posylek [12], are not considered in this paper.

ANALYTICAL SOLUTIONS

Groundwater is a concern of many engineering disciplines so analytical solutions to a large number of underground water flow models have been calculated . A description of those which have received most use can be found in text books on groundwater such as that by Raudkivi and Callander [13]. The mining industry has chiefly exploited solutions which deal with the flow of water to a well in an aquifer. Singh and Atkins [14] have listed four particular solutions due to Theis [15], Jacob and Lohman [16], Hantush and Jacob [17] and Hantush [18] and have explained how they are applied to mining problems. Each of these solutions is based on a model of a well fully penetrating a horizontal saturated aquifer which is assumed to be homogeneous, isotropic and of infinte extent.

Confined Aquifers

In the first model [15] the aquifer was confined above and below by impermeable layers and fully penetrated by a well of radius r_w as illustrated in figure 1. Initially the water level in the well and the piezometric surface in the aquifer were constant and equal to a value h_o but as pumping proceeded at a constant rate Q both fell.

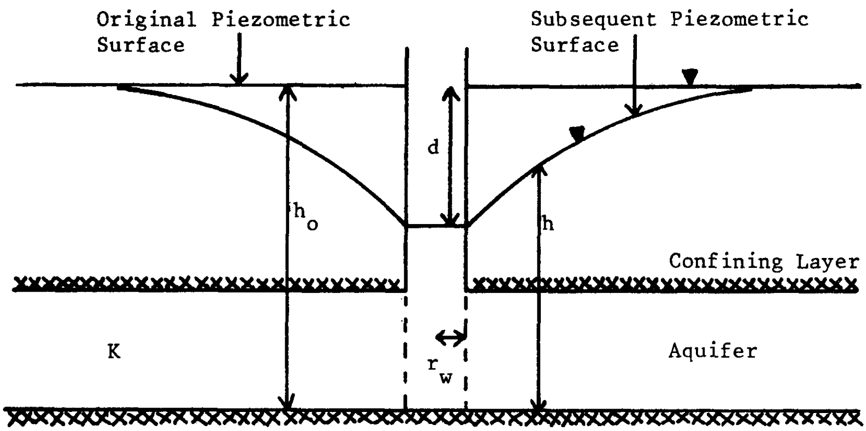

Figure 1 A Well in a Confined Aquifer

The flow of water through the aquifer and into the well was taken to be horizontal and time varying so that a two dimensional version of equation (13) had to be solved. The value of the piezometric head as a function of position and time was obtained by reference to an analogous heat flow problem and was given by

$$h_o - h = \frac{Q}{4\pi T} W(u) \qquad (15)$$

where $$u = \frac{r^2 S}{4Tt}. \qquad (16)$$

The transmissivity T and storage coefficient S of the aquifer replaced the conductivity and specific storage in the solution. W(u) is the Theis well function defined by an integral which requires numerical evaluation and which has been tabulated for a range of values.

The model solved by Jacob and Lohman [16] was indentical to the Theis model except that the pumping rate was not constant but varied in such a way that the water drawdown d in the well remained constant. Solution of the problem, again by heat analogy, gave the pumping rate as a function of time according to the equation

$$Q = 2\pi T d G(\alpha) \quad (17)$$

where $$\alpha = \frac{Tt}{S r_w^2} \quad (18)$$

$G(\alpha)$ is the Jacob-Lohman well function given by another integral requiring numerical evaluation for which tabulated values are available. Both the Theis and Jacob-Lohman well functions have assymptotic approximations capable of direct evaluation for large values of time.

Leaky Aquifers

In the models of Hantush and Jacob [17] and Hantush [18] the stratum above tha main aquifer was given a small conductivity and thus formed a semi-confining layer. Above this was a further aquifer which was able to recharge the lower or leaky aquifer through the semi-confining layer. The well was lined for most of its depth and tapped only the lower aquifer as illustrated in figure 2.

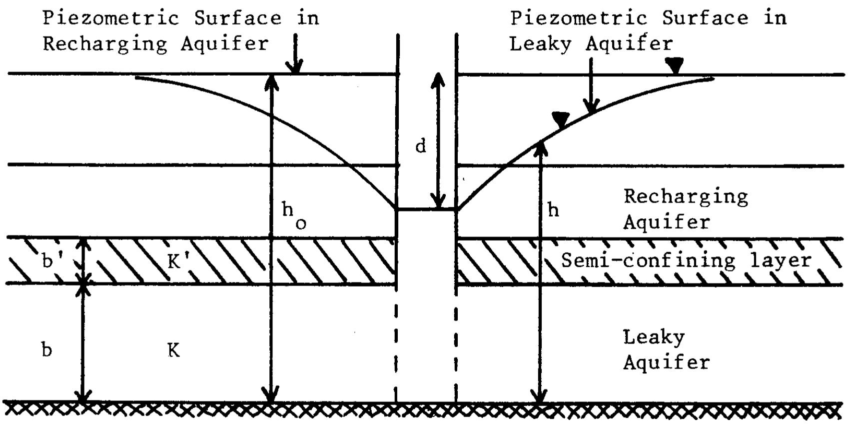

Figure 2 A Well in a Leaky Aquifer

Initially the piezometric levels in the two aquifers were constant and equal to a valued h_o so that no water flowed between them. As pumping proceeded the water level in the well and the piezometric level in the leaky aquifer both fell but the piezometric level in the recharging aquifer was assumed to stay constant. Consequently the head loss across the semi-confining layer was simply equal to the drawdown in the leaky aquifer.

The flow of water through the leaky aquifer and into the well was taken to be horizontal and time varying. The use of a three dimensional equation to simulate vertical flow from the upper aquifer was avoided by applying the mass conservation principle to a volume of leaky aquifer spanning the entire aquifer thickness. The vertical flow into this volume was given by applying Darcy's Law to the semi-confining layer. The mathematical statement of the requirement that the horizontal and vertical flow into the volume of aquifer together equalled the rate of storage was

$$b \underset{\sim}{\nabla}_1 \cdot \underset{\sim}{u} - \frac{K'}{b'} (h_o - h) = -bs \frac{\partial h}{\partial t} \qquad (19)$$

where b was the thickness of the leaky aquifer and b' and K' were the thickness and conductivity of the semi-confining layer. Combined with Darcy's Law equation (19) became

$$Kb \left(\frac{\partial^2 h}{\partial x^2}\right) + \left(\frac{\partial^2 h}{\partial y^2}\right) + \frac{K'}{b'} (h_o - h) = bs \frac{\partial h}{\partial t}. \qquad (20)$$

Hantush and Jacob [17] provided a solution to equation (20) for a constant pumping condition as

$$h_o - h = \frac{Q}{2\pi T} \left[K_o \left(\frac{r}{B}\right) + H\left(\frac{r}{B}, \frac{Tt}{SB^2}\right) \right] \qquad (21)$$

where B was a leakage factor defined by

$$B^2 = \frac{Kb\ b'}{K'}. \qquad (22)$$

$K_o\left(\frac{r}{b}\right)$ is a Bessel function and $H\left(\frac{r}{B}, \frac{Tt}{SB^2}\right)$ is an integral; both terms require numerical evaluation. The integral becomes small for large values of time and the steady state solution is given by

$$h_o - h = \frac{Q}{2\pi T} K_o\left(\frac{r}{B}\right). \qquad (23)$$

Tabulated values of the Bessel function are available.

Hantush [18] gave a solution to equation (20) for a condition of constant drawdown at the well as

$$Q = 2\pi\ t\ d\ G(\alpha, \frac{r_w}{B}) \qquad (24)$$

where $\alpha = \frac{Tt}{Sr_w^2}$ (25)

$G(\alpha, \frac{r_w}{B})$ is the Hantush well function which is a complicated expression for which numerical values have been tabulated. Approximations capable of direct evaluation exist for small and large values of time.

Well Models Applied to Mine Water Flow

The application of well models to predict water flow into mines has been described by Singh and Atkins [14]. The basis of the application is to find the smallest circle which can completely enclose the mine plan and to imagine a well drilled through its centre. The mineral deposits of an underground mine are assumed to lie within a thick aquifer which the well completely penetrates. The quantity of water flowing into the mine is approximately found by calculating the rate at which the well would have to be pumped to lower than the piezometric surface of the aquifer to below the circle enclosing the mine. The required drawdown is illustrated in figure 3. The pumping rate is given by one of the well solutions already described in which the well radius, if required, is set equal to the mine radius.

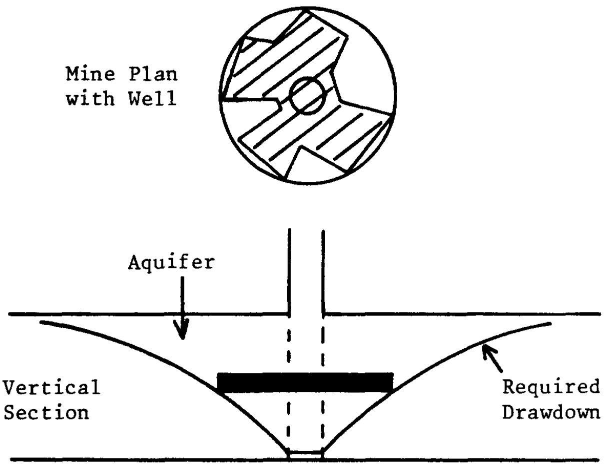

Figure 3 Imaginary Mine Dewatering

The limitations of such an approach are as follows;

1) The conceptual model of dewatering an aquifer does not properly apply to the majority of underground mining operations in which the workings are protected from water laden aquifers by intervening strata. Under such conditions the aquifer is left saturated and the rate of water leakage into the mine is chiefly determined by the properties of the protective layer.

2) The well models are used beyond the limits of their validity. All four solutions were obtained on the assumption that the piezometric surface remained above the top of the main aquifer but in the mining application it falls within the aquifer. Under such conditions the solutions are no longer valid.

3) No account is taken of the shape of the mine or of the aquifer. The mine plan is replaced by a circle of fixed size and the aquifer is assumed to be infinitely large.

4) The aquifer is assumed to be homogeneous, isotropic and horizontal.

Extensions of Well Theory

Well theory and its application to mining problems have been developed by various workers in order to describe mining operations more closely. Hantush has examined many more well models including one in which the aquifer is circular with a finite radius [18] and another in which the well only partially penetrates the aquifer [19]. Dudley [20] used the work of Hantush to model large underground openings and devised a method for approximating tunnels by cylindrical wells. He also modified the theory to describe gradual construction by allowing the size of the well to vary. Singh, Atkins and Aziz [21] have reviewed models which extend well theory to describe flow into a gallery, a trench, a tunnel and a system of two wells. The modifications of well theory still admit only very simple shapes and boundary conditions and no analytical model has been able to describe the heterogeneity and anistropy which characterise real rock strata.

NUMERICAL SOLUTIONS

Numerical techniques used in conjunction with high speed digital computers enable solutions to be found to complex groundwater problems which are beyond the scope of analytical methods. More realistic boundary shapes can be considered, boundary conditions can include time dependence and other complications, material properties can be allowed to vary in space and time and the restriction of isotropy can be removed. Against these advantages the major disadvantage of numerical methods is that they require the use of a computer whereas a hand calculator and a set of well functions suffices to find an analytical solution.

Numerical solutions to mine water problems are at best only as a accurate as the physical data which define the governing equation and boundary conditions. These data include the properties of the rock throughout the region of flow and the positions and pressures of surface and underground water sources. In practice the required information is not completely or accurately known and it may be necessary, as Owili-Eger [22] found, to supplement sparse field data with text book values. Lack of data also limits the extent to which a numerical solution can be verified by field tests but such a solution is almost certain to be more accurate than an analytical solution.

NUMERICAL TECHNIQUES

In order to solve a mine water model the governing equation must be solved within the domain of flow subject to appropriate boundary conditions defined along the entire boundary. The three basic numerical techniques available to solve this type of partial differential equation are the finite difference method, the finite element method and the boundary element method. They can be used separately or in conjunction with each other, the choice of method depending upon the details of the governing equation, the flow domain and the boundary conditions. This becomes clear on considering the possible methods in detail.

The Finite Difference Method

In the finite difference method a fixed grid is superimposed on to the domain and the equation of flow is solved only at the intersection points of the grid. Partial derivatives are replaced by an appropriate combination of the fluid heads at neighbouring grid points and the differential equation is approximated by a system of linear algebraic equations. This system is solved with standard computer routines. Flow rates are approximately calculated from the differences between fluid heads at neighbouring points. The accuracy of the solution can be improved by using a finer grid although this increases computing time.

The finite difference method can be used to solve the most complex partial differential equation but its reliance on a regular grid causes difficulties when irregular boundary shapes must be represented. A more detailed account of the method can be found in Ames [23]

The Finite Element Method

In the finite element method the domain is partitioned into a large number of sub-domains called finite elements. Within each element the local form of the solution to the governing equation is approximately represented by a linear or higher order polynomial with unknown coefficients equal to the values of the expression at specific points or nodes on the element. Mathematical manipulation of the governing equation generates a system of linear equations in the nodal values which is solved using computer techniques. Flow rates are approximately calculated from the partial derivatives of the polynomial approximations. Accuaracy can in general be improved by taking a higher order polynomial or smaller elements.

Finite elements can be made to fit any boundary reasonably closely and can vary in size throughout the domain to match the needs of different regions of solution. Care is needed in their definition to ensure that the whole domain is covered without gap or overlap. A general account of the finite element method can be found in Mitchell and Wait [24] or Hinton and Owen [25].

The Boundary Element Method

A rather different approach is given by the boundary element method where the governing equation is first redefined as an integral equation which has to be evaluated only around the boundary of the domain. This equation is solved numerically to give the undefined flow rates and fluid heads on the boundary. Details of the flow inside the domain can be obtained at a later stage.

The boundary is divided into elements and within each the undefined variable is approximately represented by some chosen form with unknown coefficients equal to its values at boundary nodes. The boundary integral equation is reduced to a system of linear equations in the nodal values and this is solved with standard computer routines. Various methods for transforming a partial differential equation within a bounded domain into a boundary integral equation have been described by Jaswon and Symm [26] and Brebbia, Telles and Wrobel [27].

Because the boundary element method reduces the number of dimensions under consideration by one its elements are less complicated than finite elements while retaining their flexibility. However the range of equations which can be solved with the boundary element method is quite limited and consequently the method has not been used to solve realistic mine water models. In particular it does not cope adequately with equations containing a heterogeneous conductivity and Brebbia [28] has suggested that under such conditions the domain of solution should be divided into a series of regions each of which is homogeneous. Clements [29] has applied the boundary element method to a partially heterogeneous problem with an isotropic conductivity varying in one direction only. Watson and Brown [30] have used the method to model water flow through anisotropic homogeneous rock.

Time Dependent Equations

All three numerical methods can be used to solve a transient equation. The time over which solution is sought is divided into small intervals separated by grid times and derivatives are replaced by the differences between values at consecutive times. The equation is solved only at grid times and in the simplest form this leads to solution of a series of 'quasi-steady' problems.

WATER RESOURCE MODELS

Many models similar to the analytical well models already described, but with more complicated boundary conditions, have been developed and solved with numerical techniques by workers within the water resources industry. These models have simulated flow in aquifer systems by two dimensional horizontal approximations with vertical flow incorporated through simple relationships. Two separate uses of a water resources approach to the modelling of mine water have been reported.

A Gravity Well Model

Schubert [31] used a numerical model developed by Trescott, Pinder and Larson [32] to simulate the removal of water from an aquifer lying above a disused coal mine into a lower aquifer by gravity wells as illustrated in figure 4. He suggested that such dewatering would decrease the amount of water flowing through the mine and thereby reduce the associated pollution. The model solved a finite difference approximation of the transient equation for two dimensional horizontal flow given by

$$\frac{\partial}{\partial x}\left[K_{xx}\, b\, \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yy}\, b\, \frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + W(x,\ y,\ t) \qquad (26)$$

in which W(x, y, t) was the volumetric flux of water recharge and withdrawal per unit surface area. This was similar to the equation (20) used by Hantush and Jacob [17] and Hantush [18] to simulate flow in a leaky aquifer with the important difference that the conductivity was anisotropic and partially hetergeneous. Full heterogeneity was precluded because the principal axes of conductivity were assumed to lie in fixed directions throughout the aquifer.

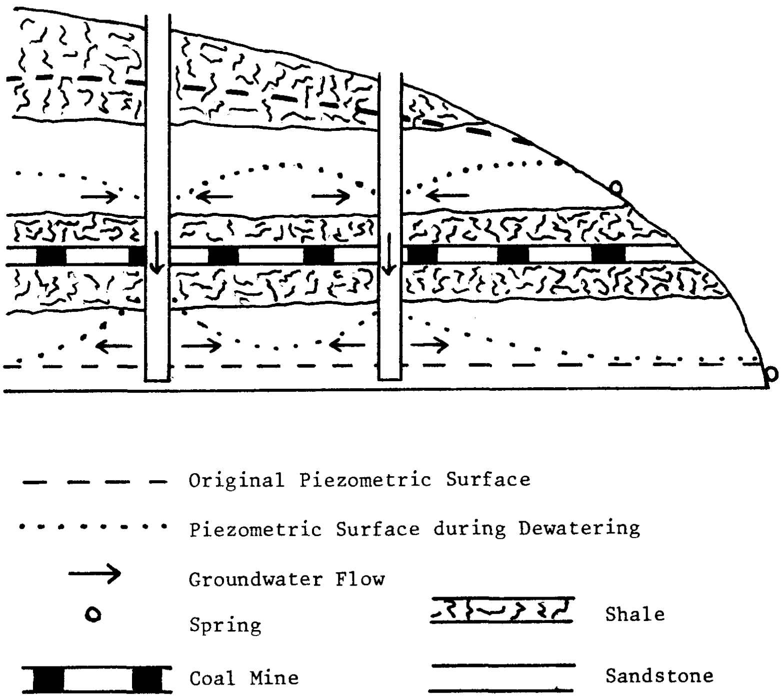

Figure 4 Gravity Well Dewatering of an Aquifer

Flow was simulated only in the source aquifer above the mine because the lower aquifer was assumed to be sufficiently thick and permeable to transmit all the water discharged into it. Darcy's Law was taken to govern the discharge of water through the shale roof of the mine and its rate was therefore determined by the hydraulic heads at the source aquifer and the mine and by the thickness and vertical conductivity of the roof. The hydraulic potential of the roof was set equal to its elevation or to the height of a mine pool where one existed. Boundaries where the aquifer outcropped were represented by a constant potential and no flow conditions applied where the aquifer thinned to zero.

Schubert used his model for a specific mine and callibrated it against the flow which existed in the absence of wells by varying the conductivities and recharge rates. He selected those values which gave the most realistic potential surface, spring discharge rates and roof leakage rate. He simulated well dewatering by increasing the discharge rate of each well until the water level inside it approached the base of the aquifer. The simulation was used to predict the reduced rate of leakage through the mine roof under dewatering conditions.

A Large Aquifer Model

Szilagyi, Heinemann and Bogardi [6] took a similar approach to simulate the effect of water withdrawals by mining operations on the piezometric level in a large aquifer. Their model described horizontal water flow within the aquifer by a finite difference approximation of essentially the same two dimensional transient equation (26) that Schubert used in his model. The area modelled was large: complete mines were represented by a few grid squares with known withdrawal rates and exposed regions of the aquifer were defined as grid squares with known infiltration rates. Outcropping boundaries of the aquifer were assumed to be at known pressures while the remaining parts of the boundary had no flow conditions applied.

The model was used to simulate a part of the karstic aquifer of the Hungarian transdanubian mountain. Callibration was achieved by altering the conductivities and some of the boundary conditions until the predicted water level for selected years conincided with observation. Application of the model to an entire twenty year period then gave simulated water levels and natural discharge rates which were generally similar to observed values. The model was to be used subsequently to predict the effect of different measures which could be taken to improve conditions within the aquifer.

MINE WATER MODELS

A fundamentally different approach to that described for water resource models has been adopted by some workers to simulate directly the rate of water flow into an operating coal mine. Water resource models simulate horizontal flow within an aquifer and include vertical flow by means of simple expressions describing water recharge from sources and discharge to sinks outside the plane of the aquifer. Mine water models simulate vertical flow directly either by resorting to a three dimensional governing equation or by using a two dimensional approximation within a vertical cross section. Three mine water models have been reported.

The Pennsylvania Model

Owili-Eger and Manula [33] have developed a two dimensional transient model, sometimes known as the Pennsylvania model, to simulate the movement of water and air in the saturated and unsaturated zones above an underground mine and to predict water flow rates into operating sections of the mine. A detailed description of the model has been given by Owili-Eger [22] and reproduced by Manula and Owili-Eger [34]. The two phase flow of air and water was simulated by defining a separate governing equation for each phase. The degree of water saturation, varying between zero and one, appeared in both equations and the two were coupled by expressing the difference between the air and water pressures as a known capillary pressure. The resultant single equation for the water pressure was rather involved and is not reproduced here. The rock was allowed to exhibit different permeabilities to air and water dependent upon the level of saturation. Each was expressed as the product of a base permeability common to both phases and an air or water relative permeability. The base permeability was anisotropic and partially heterogeneous but its principal axes were assumed to be vertical and horizontal throughout the domain of the model. The relative permeabilities were scalar multipliers which, along with the capillary pressure between air and water, were functions of water saturation.

A finite difference approximation coupled with finite element analysis was used to give a numerical solution to the model. Flow was assumed to be horizontal in a principal reservoir and vertical in every other stratum. Values which had to be defined for each finite element included base permeability, water saturation, two relative permeabilities and capillary pressure. Boundary data required at the ground surface included precipitation, temperature, atmospheric pressure, all as functions of time, and the positions of any rivers or lakes. The mine was assumed to be at a constant pressure and the remainder of the boundary of the vertical cross section had a no flow condition applied.

The model was applied in a case study to three operating sections of a coal mine in Pennsylvania. The mine opening was assumed to be static during the one year of simulation but the model could describe moving openings. A rather large amount of input data was required and it was found necessary to supplement field data with text book values. Simulated mine drainage quantities for each month throughout one year were compared with recorded quantities and a maximum difference of 22.0% was noted while the average was 5.8%.

A Coupled Subsidence and Water Flow Model

Girrens et al. [35] described a coupled model, in the early stages of development when reported, to predict the water flow and subsidence associated with mining operations. The simulation was three dimensional and transient. Permeability was allowed to be anisotropic but only partially heterogeneous because its principal axes were assumed to lie along the three fixed cartesian axes. An existing finite element code designed for the analysis of heat flow problems was used to give a numerical solution.

Confined and unconfined flow could be simulated, the latter requiring the position of a free surface to be calculated by applying the conditions that hydraulic potential was equal to elevation along it and that water flow was zero zcross it. Usually such a calculation is achieved by iteration but Girrens et al. adopted a direct method in which the permeability in the governing equation was a function of the position of the free surface.

The model was used to simulate radial flow to a well, matching the analytical solutions of Theis [15] and Hantush [19] fairly closely. It was also used to simulate unconfined flow through a porous rectangular dam with isotropic homogeneous permeability, giving flow rates into and out of the dam which differed by only 1.5%. The model was finally applied to a simple representation of flow into a longwall extraction based on the subsidence damage predicted by Singh and Kenorski [36]. A homogeneous permeability was arbitrarily assigned throughout a zone of damage, illustrated in figure 5, while the undamaged zone had a zero permeability. None of the simultations were tested against field data.

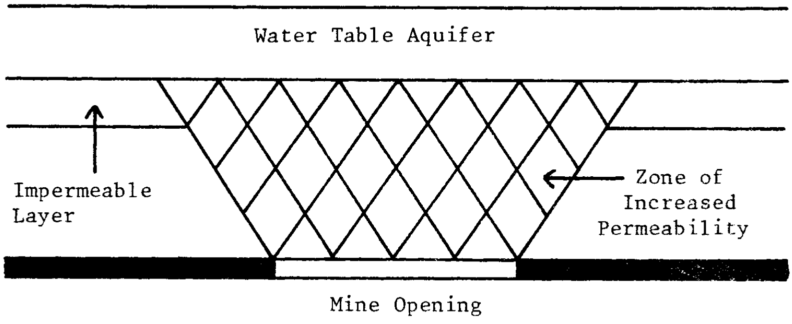

Figure 5 Effects of Longwall Mining Damage

An Acid Pollution and Water Flow Model

Ricca and Hemmerich [37] described a mathematical model for predicting the quantity of water flowing into an underground mine system together with the acid load polluting the discharged water. The details of the simulations have been more fully reported by Schumate et al. [38]. The quantity prediction part of the model used the previously established Stanford Watershed Model which simulated the hydrological behaviour of a basin. This model used independently derived empirical relationships rather than Darcy's Law to describe the movement of groundwater. Flow into the mine was simulated by incorporating a delay between the times when water entered and left an aquifer. The watershed model required 38 input parameters, 12 of which were measurable while 11 were arbitarily assigned and the remaining 15 could be adjusted in callibration procedures. A further 32 parameteers describing the mine and its polluting properties were needed to run the complete mine water quantity and quality prediction model. Permeability was not one of the input parameters.

The full model was applied in one case study to a watershed within West Virginia. Initially only the watershed part of the model was used and adjusted until the simulated stream flows matched recorded values as closely as possible. Although the overall correlation was fairly good individual stream flows were not well reproduced. Further extensive adjustment was necessary when the full mine model was applied in order to make the calculated mine flow rates and acid loads match observed values. Gross trends were fairly well simulated but over a year the total flow was underestimated while the acid pollution was overestimated. The model required a large amount of input data and although the case study was chosen because much of the necessary information was available many values still had to be estimated.

METHANE MODELS

Several mathematical simulations of methane movement in strata near coal mining operations have been reported. The flow of methane in porous media is governed by Darcy's Law and a mass conservation equation, however the statement of mass conservation for methane is complicated by the compressibility of the gas and by the adsorption of methane on to the coal. As a result models which only simulate methane flow are not directly applicable to water.

A Methane and Water Model

Price and Abdallah [39] have developed a model to simulate the two phase flow of methane and water in a coal stratum. A two dimensional transient equation described the horizontal movement of each fluid and the two equations were coupled in the same manner described for the Pennsylvania model. The equation for methane flow possessed a source term, like that in equation (26), to simulate desorption and adsorption. The coal was assigned a heterogeneous and isotropic base permeability and two relative permeabilities for the two phases.

The finite difference method based on an irregular grid was used to give a numerical solution to the model. The outer boundaries of the coal bed away from the mine working had no flow conditions applied and the mine was represented by a moving boundary with one complete grid square of coal being replaced by void in each time step. The model had not been tested against a real coal mine.

BRITISH LONGWALL MINING

A primary purpose of this review is to establish whether any of the mathematical models currently available can adequately describe conditions near a British longwall coal operation and consequently predict the rate of water flow into the workings. British longwall mines lie at typical depths of around 500 metres, with an overall range from less than 100 metres to more than 900 metres. The vast majority of works are not therefore affected by surface water but many are infiltrated by water from heavily soaked aquifers. Watson [40] and Whittaker and Aston [41] concluded that even under the North Sea all water reaching mine workings originated in aquifers and not the sea itself. Such sources can provide extremely large quantities of water as instanced by the flooding of the Wistow mine in the Selby coalfield [42] when 80 million litres (18 million gallons) of water from a limestone aquifer poured through a longwall face at the rate of 190 l/s (2500 gallons a minute).

The flow of water to longwall workings is highly influenced by the effects of mining subsidence. As a longwall face progresses the strata above and behind it are allowed to cave into the void created by mining. As they do so they crack and separate thus increasing the local vertical and horizontal permeability. There is no precise information available about the nature of these permeability changes but experimental results reported by Whittaker, Singh and Neate [43] indicate that the effect is rather too large to ignore. Anisotropic properties were not measured but it was found that the average permeability of rock strata above a coal seam increased by a factor between 40 and 80 as the face passed below the measurement point.

Numerical Models Applied to Longwall Mining

A fundamental requirement of any mathematical model predicting water flow into a British longwall working is that it must be able to simulate the high degree of local heterogeneity which results from caving. This effect is a feature of the vertical direction. The water resource models [6] and [31] and the methane and water model [39] were horizontal simulations and could not therefore describe complex vertical phenomena. The acid pollution model [37] used empirical relationships rather than Darcy's Law to describe fluid flow so a heterogeneous permeability could not be modelled because it was not one of the input parameters.

The Pennsylvania model [33] accepted the heterogeneous permeability necessary to describe the effects of longwall mining damage. The inclusion of the unsaturated flow regime in order to describe workings at depths of as little as 60 metres made the flow equations rather involved and required the input of a large amount of data. A consideration of unsaturated flow is an unnecessary complication in any simulation of a British longwall mine and a model which solved only the saturated flow equations could be more easily and reliably modified to examine various heterogeneous permeability effects and would demand less input data.

In the subsidence model [35] Girrens et al attempted to deal directly with longwall mining subsidence but they used only a simple model based on the qualitative work of Singh and Kendorski [36]. Unfortunately there has been no subsequent report of their work.

A limitation common to the Pennsylvania model [33] and the subsidence model [35] is the assumption that the principal axes of permeability are vertical and horizontal throughout the domain of solution. This is unlikely to be true for the inclined, folded and fractured strata which surround many mine workings.

CURRENT AND FUTURE WORK

Work is currently being conducted at Nottingham University to develop a mathematical model which is specifically designed to describe underground water flow near British longwall coal operations. An important part of this work will be to investigate the permeability changes reported by Whittaker, Singh and Neate [43] and to determine how they affect the flow of water stored in an aquifer. A simple model of a single longwall face surrounded by homogeneous and isotropic rock has already been developed and solved by the finite difference, finite element and boundary element techniques in order to compare the suitability of each method for solving groundwater flow problems. The findings of this work are to be reported.

Future work will be concerned initially with modelling the effects of the permeability changes caused by mining damage from a single longwall face within horizontal strata. Once these effects have been well understood the model will be extended to simulate inclined strata and to deal with multiple faces in the same and different coal seams.

CONCLUSIONS

Mathematical models of groundwater flow can provide a useful predictive tool for the mining industry and several analytical and numerical models are already in use. Analytical models suffer severe limitations because they can only describe homogeneous and isotropic media and simple boundary shapes. Numerical models are more flexible but they require a substantial amount of data to run them and to verify their results in test cases. Current numerical models use the finite difference and finite element solution methods because the boundary element method cannot easily simulate heterogeneous effects. Only two published models are able to simulate complicated vertical phenomena and neither has been used to investigate in detail the permeability changes due to longwall coal mining subsidence. No model can simulate rock strata in which the principal axes of permeability are inclined or variable. More work is needed to develop the modelling of water flow near longwall coal workings and this has started at Nottingham University.

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